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Kinematic Control
of Robot With
Degenerate Wrist

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NASA

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SUMMARY

A popular method of controlling a robot arm is to use kinematic resolved-rate equations, which allow an operator to dynamically control the arm by using visual feedback. In using resolved-rate equations, an operator commands robot hand velocity, which is resolved into joint angle rates to cause the commanded motions. By visual feedback, the operator adjusts these commands.

The robot wrist consists of three rotational joints in the robot arm. In this paper, the wrist loses a degree of freedom (degenerate condition) when the second wrist joint is at 0° so that the first and third wrist joints have colinear rotational axes. Methods are analyzed for controlling the robot hand at or near this degenerate wrist condition.

The resolved-rate equations become indeterminant when the robot wrist is degenerate, and slightly away from this condition, very large joint angle rates are calculated. From a practical viewpoint, the joints cannot rotate faster than some maximum speed. Therefore, resolved-rate control with rate limiting is examined. Whenever a computed joint angle rate exceeds a specified maximum value, the maximum value is used. It is shown that resolved-rate control with rate limiting produces numerous unexpected responses in the degenerate wrist region. The generalized matrix inverse solution to the resolved-rate equations can also produce unwanted responses to commanded inputs.

A new method is introduced to control the robot hand in the region of the degenerate wrist. The method uses a coordinated movement of the first and third joints of the robot wrist to locate the second wrist joint axis for movement of the robot hand in the commanded direction. This method does not entail infinite joint angle rates or cause extraneous hand movements; however, there is a brief delay for the coordinated movement (which is accomplished in an optimal manner).

INTRODUCTION

Controlling individual joints in a robot arm to accomplish a complex task is difficult, especially if time to complete the task is critical or if part of the operator's attention is needed elsewhere. A more natural approach is for an operator to command the motion of the robot hand and then automate the requisite coordination of the individual joints in the arm. An operator can use kinematic, resolved-rate equations (ref. 1) to dynamically control a robot arm by watching its response to commanded inputs. The operator views the robot hand, decides that he wants it to move in a certain direction, and deflects a controller. The robot hand then moves accordingly with a velocity proportional to the amount of deflection of the controller. Commanded hand velocities are transformed (resolved) into requisite movements (velocities) of the individual joints in the robot arm to effect the commanded hand motion.

Three rotational joints in the robot arm constitute the robot wrist. Whenever any two of these joints have their rotational axes colinear, the wrist loses a degree of freedom and is said to be degenerate. In this condition, the resolved-rate equations are indeterminant. In close proximity to the degenerate condition, calculated

joint angle rates can exceed maximum operational limits. The emphasis in reference 2 is on predicting and avoiding such degenerate regions; however, this reduces the working regions of the robot wrist and requires the arrangement of tasks to keep the wrist out of the degenerate orientation. The intent in this paper is to continue controlling the robot hand at (or near) the degenerate wrist condition to allow more flexible movement geometry.

SYMBOLS

A_{i-1}^i homogeneous transformation matrix from coordinate system i to $i-1$

a_i perpendicular distance between Z_{i-1} and Z_i

G L times sign of $\dot{\theta}_6$ in equation (16)

i integer to indicate different axis systems and associated parameters

J Jacobian matrix

k integer corresponding to value used in optimal coordinated movement

L maximum joint angle rate limit

M L times sign of $\dot{\theta}_4$ in equation (15)

O_i origin of joint i axis system

$\vec{q}'$ vector to point Q

R_{i-1}^i rotational transformation matrix from coordinate system i to $i-1$; 3×3 matrix in upper left corner of A_{i-1}^i

R_i^{i-1} inverse (transpose) of R_{i-1}^i

R_6^i rotational transformation matrix from coordinate system i to hand coordinate system (see eq. (11))

r_i distance between coordinate systems $i-1$ and i along Z_{i-1}

t time

$\vec{v}$ linear velocity of robot hand

$\vec{v}_c$ commanded linear velocity of robot hand

v_X, v_Y, v_Z magnitude of $\vec{v}_X$, $\vec{v}_Y$, and $\vec{v}_Z$, respectively

$\vec{v}_X, \vec{v}_Y, \vec{v}_Z$ linear velocity components of robot hand (in hand axis system)

X_f, Y_f, Z_f forearm axis parallel to X_3, Y_3, and Z_3, respectively

X_i axis directed along common normal between Z_{i-1} and Z_i

Y_i axis directed to complete right-hand axis system with X_i and Z_i

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Z_i axis of rotation of joint $i + 1$

α_i angle between Z_{i-1} and Z_i, measured positively about positive X_i

β angle between $\vec{\Omega}_c$ and X_6

δ_i candidate variation in θ_4 for coordinated movement

δ_k minimal variation chosen for θ_4 in coordinated movement

θ_i joint angle with initial value corresponding to position of robot arm in figure 2

θ_i' joint angle between X_{i-1} and X_i, measured positively about positive Z_{i-1}

λ_i value for θ_6 corresponding to value ψ_i for θ_4 in coordinated movement

λ_k optimal λ_i used in coordinated movement

ψ_i candidate value for θ_4 in coordinated movement

ψ_k optimal ψ_i used in coordinated movement

$\vec{\Omega}_c$ commanded rotational velocity of robot hand in plane of hand axes X_6 and Y_6

$\vec{\omega}$ resultant rotational velocity of robot hand (in hand axis system)

$\vec{\omega}_c$ commanded rotational velocity of robot hand (in hand axis system)

$\omega_X, \omega_Y, \omega_Z$ magnitude of $\vec{\omega}_X$, $\vec{\omega}_Y$, and $\vec{\omega}_Z$, respectively

$\vec{\omega}_X, \vec{\omega}_Y, \vec{\omega}_Z$ rotational velocity components of robot hand (in hand axis system)

$\vec{\omega}_{1,2,3}$ rotational velocity of robot hand caused by rotation of joints 1, 2, and 3 (in hand axis system)

$\vec{\omega}_{4,5,6}$ rotational velocity of robot hand caused by rotation of three wrist joints 4, 5, and 6 (in hand axis system)

Subscript:

c   value commanded by operator

Abbreviations:

E   elbow of robot arm

ES   elbow-to-shoulder length

H   hand of robot arm

HW   hand-to-wrist length

N	neck of robot arm
NO	neck-to-base length
O	base of robot arm
S	shoulder of robot arm
SN	shoulder-to-neck length
W	wrist of robot arm
WE	wrist-to-elbow length

Use of a dot over a symbol indicates the first derivative with respect to time.

ANALYSIS

Resolved-rate control enables an operator to effectively issue commands (such as "go forward" or "pitch up") to the robot hand by commanding translational and rotational velocities in the hand's own axis system. These commanded velocities are resolved by transformation matrices into joint angle rates along the robot arm.

Joint Axis Systems and Transformation Matrices

Consecutive joint axis systems in robotic manipulators can be related by the Denavit-Hartenberg parameters (ref. 3). For rotational joints (fig. 1), these parameters consist of three constant parameters a_i, r_i, and α_i and a variable joint angle θ_i'. By definition, joints always rotate about their Z-axis. The Y_{i-1}-axis and Y_i-axis (not shown) complete right-handed coordinate systems. The parameter r_i plays the role of s_i in reference 4 and d_i in reference 3.

The homogeneous transformation matrix (based on fig. 1) from coordinate system i to coordinate system $i - 1$ is (ref. 4 or 5, for example)

$$A_{i-1}^i = \left[\begin{array}{ccc|c} \cos\theta_i' & -\cos\alpha_i \sin\theta_i' & \sin\alpha_i \sin\theta_i' & a_i \cos\theta_i' \\ \sin\theta_i' & \cos\alpha_i \cos\theta_i' & -\sin\alpha_i \cos\theta_i' & a_i \sin\theta_i' \\ 0 & \sin\alpha_i & \cos\alpha_i & r_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad (1)$$

Let the location of a point Q with respect to the coordinate system (X_i, Y_i, Z_i) be described by the vector $\vec{q}_i$. Then, the location of Q from coordinate system $(X_{i-1}, Y_{i-1}, Z_{i-1})$ is the vector $\vec{q}_{i-1}$, where

$$\left\{\frac{\vec{q}_{i-1}}{1}\right\} = A_{i-1}^i \left\{\frac{\vec{q}_i}{1}\right\} \qquad (2)$$

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in which A_{i-1}^i accounts for both rotation and displacement of (X_i, Y_i, Z_i) with respect to $(X_{i-1}, Y_{i-1}, Z_{i-1})$. However, if one is only interested in the components of $\vec{q}_i$ in directions parallel to $(X_{i-1}, Y_{i-1}, Z_{i-1})$, such as velocity, or if adjacent axis systems have coincident origins, then it is sufficient to compute

$$\vec{q}_{i-1} = R_{i-1}^i \vec{q}_i \qquad (3)$$

where

$$R_{i-1}^i = \begin{bmatrix} \cos\theta_i' & -\cos\alpha_i \sin\theta_i' & \sin\alpha_i \sin\theta_i' \\ \sin\theta_i' & \cos\alpha_i \cos\theta_i' & -\sin\alpha_i \cos\theta_i' \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix} \qquad (4)$$

Figure 2 is a modification of a figure in reference 4 and illustrates a robot arm and joint axis systems. The Denavit-Hartenberg parameters and joint angle limits assumed in reference 4 (and here) are shown in table I. Notice that θ_i' is related to another joint angle θ_i (unprimed). The joint angles θ_i ($i = 1, 2, \ldots, 6$) are referenced to the initial position of the robot arm in figure 2.

The transformation matrix A_{i-1}^i (in terms of θ_i, $i = 1, 2, \ldots, 6$) is given in reference 4 and repeated in appendix A. The rotational matrix R_{i-1}^i is simply the 3×3 submatrix in the upper left-hand corner of A_{i-1}^i.

The axis for the robot hand may be located wherever desired for convenience, for example, near the tip of the robot hand, at the robot hand mounting or, as in reference 4 and here, at the origin of the wrist axis system (HW = 0). In the sequel, commands of rotational and translational velocity to the robot hand are expressed in terms of joint angle rates.

Equations for Resolved-Rate Control

As indicated in figure 3, an operator commands the translational velocity

$$\vec{V}_c = \begin{Bmatrix} V_X \\ V_Y \\ V_Z \end{Bmatrix}_c \qquad (5)$$

and the rotational velocity

$$\vec{\omega}_c = \begin{Bmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{Bmatrix}_c \qquad (6)$$

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of the robot hand (in the hand axis system). On the basis of visual and probably force feedback, the operator varies these inputs. Equations which relate the operator's inputs for the hand motion (velocity) to joint angle rates in the robot arm are based on the transformation matrices between joint axis systems.

Translational Velocity of Robot Hand

As in reference 4, it is assumed that $HW = 0$ and that translational velocity of the robot hand is attributed entirely to the first three joint angle rates $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\theta}_3$. Thus,

$$\vec{V} = J \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{Bmatrix} \tag{7}$$

where J is an appropriate Jacobian matrix (ref. 4). Equation (7) is solved for the joint angle rates $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\theta}_3$. There are singularities (refs. 4 and 6) associated with the matrix J, but these are not considered here.

Rotational Velocity of Robot Hand (In Hand Axis System)

The rotational velocity of the robot hand is

$$\vec{\omega} = \vec{\omega}_{1,2,3} + \vec{\omega}_{4,5,6} \tag{8}$$

where

$$\vec{\omega}_{1,2,3} = R_6^0 \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{Bmatrix} + R_6^1 \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{Bmatrix} + R_6^2 \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{Bmatrix} \tag{9}$$

is the rotational velocity caused by the first three joints in the robot arm, and

$$\vec{\omega}_{4,5,6} = R_6^3 \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{Bmatrix} + R_6^4 \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_5 \end{Bmatrix} + R_6^5 \begin{Bmatrix} 0 \\ 0 \\ \dot{\theta}_6 \end{Bmatrix} \tag{10}$$

is the rotational velocity caused by the wrist joints. Note that the transpose (inverse) of equation (4) is used in equations (9) and (10). In equations (9) and (10),

$$R_6^i = R_6^5 R_5^4 \ldots R_{i+1}^i \tag{11}$$

is a product of rotational transformation matrices, which are obtained from the transpose of equation (4).

Rotational Speeds of Robot Wrist and Hand

Express equation (8) as

$$\vec{\omega}_{4,5,6} = \vec{\omega}_c - \vec{\omega}_{1,2,3} \tag{12}$$

where $\vec{\omega}_c$ denotes the commanded value for $\vec{\omega}$. The joint angle rates $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\theta}_3$ needed to compute $\vec{\omega}_{1,2,3}$ are obtained from equation (7). Hence, equation (12) gives the required $\vec{\omega}_{4,5,6}$ needed to generate $\vec{\omega}_c$. For simplicity in this analysis, $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = 0$ so that $\vec{\omega}_{1,2,3} = 0$, and equation (12) becomes

$$\vec{\omega}_{4,5,6} = \vec{\omega}_c \tag{13}$$

Wrist Speed From Commanded Hand Rotation

Equation (13) is equivalent to the three scalar equations (see appendix B)

$$\dot{\theta}_5 = (\omega_X)_c \sin \theta_6 + (\omega_Y)_c \cos \theta_6 \tag{14}$$

$$\dot{\theta}_4 \sin \theta_5 = -(\omega_X)_c \cos \theta_6 + (\omega_Y)_c \sin \theta_6 \tag{15}$$

$$\dot{\theta}_6 = (\omega_Z)_c - \dot{\theta}_4 \cos \theta_5 \tag{16}$$

Equation (14) clearly shows that there is no difficulty in computing $\dot{\theta}_5$, and equations (15) and (16) show that difficulty in computing $\dot{\theta}_4$ and $\dot{\theta}_6$ occurs only when $\sin \theta_5 = 0$. Computation of $\dot{\theta}_4$ and $\dot{\theta}_6$ when $\sin \theta_5 = 0$ is the basic problem addressed in this paper.

Computed Hand Rotational Rates

With $\vec{\omega}_{1,2,3} = 0$, equation (12) relates hand rotational rates to joint angle rates (see appendix B) as

$$\omega_X = -\dot{\theta}_4 \sin \theta_5 \cos \theta_6 + \dot{\theta}_5 \sin \theta_6 \qquad (17)$$

$$\omega_Y = \dot{\theta}_4 \sin \theta_5 \sin \theta_6 + \dot{\theta}_5 \cos \theta_6 \qquad (18)$$

$$\omega_Z = \dot{\theta}_4 \cos \theta_5 + \dot{\theta}_6 \qquad (19)$$

Of course, identities result when equations (14) to (16) are substituted into equations (17) to (19) when the commanded and actual hand rates are equal. However, equation (15) is useless in computing $\dot{\theta}_4$ when the wrist is degenerate (sin $\theta_5 = 0$), and another means of computing $\dot{\theta}_4$ is needed. Whatever values of $\dot{\theta}_4$, $\dot{\theta}_5$, and $\dot{\theta}_6$ are used in the degenerate region, equations (17) to (19) give the hand rotational rates that result from these wrist joint angle rates.

Methods for Controlling Robot Hand in Degenerate Wrist Region

Joint variables can be restricted to keep the robot wrist away from its degenerate position (sin $\theta_5 = 0$). But, the approach in this paper is to try to continue controlling the robot hand even though the robot wrist approaches or reaches its degenerate state.

Resolved-Rate Control With Maximum Rate Limit

Practically, a robot arm is limited in the speed with which its joints can move. To represent this mathematically, anytime the joint angle rate $\dot{\theta}_4$ in equation (15) has a magnitude greater than a specified operational limit L, replace $\dot{\theta}_4$ with M, which is L times the sign of $\dot{\theta}_4$ in equation (15). Likewise, when $\dot{\theta}_6$ in equation (16) has a magnitude greater than L, replace it with G, which is L times the sign of $\dot{\theta}_6$ in equation (16). Thus, as sin $\theta_5 \rightarrow 0$, equations (15) and (16) are replaced by

$$\dot{\theta}_4 = M \qquad \qquad \text{If} \quad |\dot{\theta}_4| > L \qquad (20)$$

$$\dot{\theta}_6 = \begin{cases} G & \text{If} \quad |\dot{\theta}_6| > L \\ (\omega_Z)_c - M \cos \theta_5 & \text{If} \quad |\dot{\theta}_4| > L, \text{ but } |\dot{\theta}_6| < L \end{cases} \qquad (21)$$

Substituting equations (14), (20), and (21) into equations (17) to (19) gives the resulting hand rates

$$\omega_X = -M \sin \theta_5 \cos \theta_6 + \left[(\omega_X)_c \sin \theta_6 + (\omega_Y)_c \cos \theta_6\right] \sin \theta_6 \qquad (22)$$

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$$\omega_Y = M \sin \theta_5 \sin \theta_6 + \left[(\omega_X)_c \sin \theta_6 + (\omega_Y)_c \cos \theta_6 \right] \cos \theta_6 \tag{23}$$

$$\omega_Z = \begin{cases} M \cos \theta_5 + G & \text{If } |\dot{\theta}_4| > L \text{ and } |\dot{\theta}_6| > L \\ (\omega_Z)_c & \text{If } |\dot{\theta}_4| > L, \text{ but } |\dot{\theta}_6| < L \end{cases} \tag{24}$$

In equation (24), the computed speed ω_Z is equal to the commanded speed $(\omega_Z)_c$ if $\dot{\theta}_4$ reaches its limit but $\dot{\theta}_6$ does not. If $\dot{\theta}_4$ and $\dot{\theta}_6$ both reach the limit, ω_Z only varies with θ_5.

Equations (22) and (23) in nondimensional form are

$$\frac{\omega_X}{M} = E_X + \left[\left(\frac{\omega_X}{M} \right)_c \sin \theta_6 + \left(\frac{\omega_Y}{M} \right)_c \cos \theta_6 \right] \sin \theta_6 \tag{25}$$

$$\frac{\omega_Y}{M} = E_Y + \left[\left(\frac{\omega_X}{M} \right)_c \sin \theta_6 + \left(\frac{\omega_Y}{M} \right)_c \cos \theta_6 \right] \cos \theta_6 \tag{26}$$

where

$$E_X = -\sin \theta_5 \cos \theta_6 \tag{27}$$

$$E_Y = \sin \theta_5 \sin \theta_6 \tag{28}$$

Equations (25) and (26) express the nondimensional rotational speeds ω_X/M and ω_Y/M as functions of the commanded nondimensional rotational speeds $(\omega_X/M)_c$ and $(\omega_Y/M)_c$ for wrist joint angles θ_5 and θ_6. These equations are plotted in figure 4 for $\theta_5 = 0°$ (degenerate condition) and selected values of θ_6. For other values of θ_5, simply add the results of equations (27) and (28). Suppose $(\omega_Y/M)_c = 0$. Then in figure 4(a), if $\theta_6 = \pm90°$ or $\pm270°$, the computed value ω_X/M always equals the commanded value $(\omega_X/M)_c$. Whereas, if $\theta_6 = 0°$ or $\pm180°$, ω_X/M always equals 0 regardless of the commanded speeds, and the robot hand will not respond at all to commands.

Suppose $(\omega_X/M)_c = 0$ and $(\omega_Y/M)_c = 1$. Then, for $\theta_6 = 45°$ or $-135°$ in figure 4(b), $\omega_X/M = 0.5$ and $\omega_Y/M = 0.5$. In other words, an operator commands a sole rotational speed about the Y-axis of the robot hand and, in return, gets rotational speeds about both the X-axis and Y-axis of the robot hand. As seen by a stationary observer, this coupling would cause a conical motion of the robot hand (Z_6 in fig. 2). Such motions have been observed in trial laboratory tests and were thought to result entirely from inexact Denavit-Hartenberg parameters; however, as shown here, this type of motion can result from using resolved-rate control with rate limiting. Obviously, other methods must be considered for controlling a robot hand with a degenerate wrist to avoid erratic behavior.

Generalized Matrix Inverse

There is no problem in calculating $\dot{\theta}_5$ from equation (14), and equations (15) and (16) may be solved as

$$\left\{ \begin{matrix} \dot{\theta}_4 \\ \dot{\theta}_6 \end{matrix} \right\} = \begin{bmatrix} \sin\theta_5 & 0 \\ \cos\theta_5 & 1 \end{bmatrix}^* \left\{ \begin{matrix} -(\omega_X)_c \cos\theta_6 + (\omega_Y)_c \sin\theta_6 \\ (\omega_Z)_c \end{matrix} \right\} \tag{29}$$

where the asterisk (*) denotes the generalized matrix inverse.

There is a tendency to think that the generalized inverse of a matrix is the cure for all ills caused by singularities because the inverse always exists and is unique; however, the solution formulated still may not be acceptable. In general, several factors should be kept in mind when attempting to apply the generalized matrix inverse: (1) real-time application may be hindered because it takes too long to compute the inverse; (2) the elements of the inverse are not continuous functions of the elements of the matrix which is inverted, and there are jumps when the matrix changes rank (ref. 7); and (3) for some kinematically redundant manipulators, control based on generalized matrix inverses may lead to undesirable arm configurations (ref. 8). Later, an example is given which shows that unwanted responses can be generated by equation (29).

Coordinated Movement of Robot Wrist

In figure 2, the robot wrist is degenerate because Z_3 and Z_5 are parallel, and there is a problem with the resolved-rate equations because $\theta_5 = 0°$ in equation (15). Forget about the resolved-rate equations for a moment and suppose an operator simply wants to rotate the robot hand about X_6 in figure 3. How can this be done without introducing extraneous motions of the robot hand? One way, for example, is to simultaneously make $\theta_4 = -90°$ and $\theta_6 = 90°$ by varying joints 4 and 6 at the same rates so that the robot hand does not rotate. Then, vary θ_5 with joint 5 at the desired hand rate. This is the basic idea of the coordinated movement discussed in this section. As the wrist moves away from the degenerate condition, the resolved-rate equations are again used.

Forearm axis system.- Consider the axis system $(X_f, Y_f, Z_f$ in fig. 5) which is fixed relative to the forearm. For the degenerate condition of the robot arm in figure 5, X_f, Y_f, and Z_f are aligned with X_5, Z_4, and Z_3, respectively. This axis system and the limits in table I for the wrist joint angles θ_4, θ_5, and θ_6 are shown in figure 6.

Coordinated movement.- Given that the robot wrist is in (or nearly in) the degenerate position shown in figure 5, suppose that an operator wants to rotate the robot hand about the direction indicated by $\vec{\Omega}_c$, which makes an angle

$$\beta = \tan^{-1}\left(\frac{\omega_Y}{\omega_X}\right)_c \tag{30}$$

10

with the X_6 axis. θ_4 is varied in figure 7 to make X_5 normal to $\vec{\Omega}_c$ (which places Z_4 along $\vec{\Omega}_c$). Meanwhile, θ_6 is varied at the same rotational speed in the opposite direction to θ_4 to cancel any rotation of the robot hand that would result from varying θ_4. The objective is to choose the smallest variation in θ_4 which will locate X_5 normal to $\vec{\Omega}_c$ and yet not violate the limits on either θ_4 or θ_6. There are four possibilities, depending on the value of θ_4 when the movement is initiated. In figure 8, the possible variations in θ_4 are denoted by

$$\delta_1 = \theta_6 + \beta + 90° \tag{31}$$

$$\delta_2 = \delta_1 - 180° \tag{32}$$

$$\delta_3 = \delta_1 - 360° \tag{33}$$

$$\delta_4 = 360° + \delta_2 \tag{34}$$

The coordinated movement is explained as follows: whenever θ_5 is close to $0°$ (by a specified amount), equations for the coordinated movement are used. Let

$$\psi_i = \theta_4 + \delta_i \tag{35}$$

$$\lambda_i = \theta_6 - \delta_i \tag{36}$$

for $i = 1, 2, 3, 4$ be paired candidate values of θ_4 and θ_6 for the coordinated movement. Let δ_k be the least δ_i (eqs. (31) to (34)) in absolute value such that $|\psi_i| < 135°$ and $|\lambda_i| < 270°$; that is, a coordinated movement is possible. Then,

$$\theta_4 = \psi_k \tag{37}$$

$$\theta_6 = \lambda_k \tag{38}$$

are the new target values of θ_4 and θ_6 for the coordinated movement. In this movement, which is automated, θ_4 and θ_6 are varied in unison at a selected joint angle rate to the target values. Once this coordinated movement to the intermediate orientation is completed, the robot hand rotation is accomplished by setting

$$\dot{\theta}_5 = \Omega_c (-1)^{k+1} \tag{39}$$

where Ω_c is the magnitude of $\vec{\Omega}_c$. For example, if $k = 1$, then δ_1 is the optimal variation in θ_4 in figure 8. This means that X_5 will be placed 90° counterclockwise from $\vec{\Omega}_c$, and Z_4 will be aligned with $\vec{\Omega}_c$. Hence, a positive rotation about Z_4 to make $\dot{\theta}_5$ equal to the magnitude of $\vec{\Omega}_c$ will produce the commanded rotational vector $\vec{\Omega}_c$.

With no joint limits (continuous rotational joints) on θ_4 and θ_6, δ_k always exists so that a coordinated movement is possible, but with limits on θ_4 and θ_6, there are orientations from which the movement cannot be made. In these cases, the operator may choose to control the wrist joints separately.

An operator should be given a backup option to individually control the three wrist joint rates $\dot{\theta}_4$, $\dot{\theta}_5$, and $\dot{\theta}_6$. This option bypasses mathematical singularities in wrist rotation. A disadvantage of this mode is that the operator must now think about how his changes in the wrist joint angles will affect the robot hand movement.

EXAMPLE

Figure 9 shows three sequential orientations of the robot hand axis system (X_6, Y_6, Z_6) and commanded rotational rates of the robot hand. In figure 9(a), the operator commands a rotation $(\omega_Y)_c$ about the Y_6-axis to raise the Z_6-axis to the vertical position shown in figure 9(b). In figure 9(b), the operator then commands a rotation about X_6 to tilt Z_6 as shown in figure 9(c). The robot wrist is in its degenerate position in figure 9(b). A small commanded rotational rate of 1 deg/sec is chosen for illustration.

Using Resolved-Rate Control With Maximum Rate

Table II shows the commanded hand rotational rates and the resulting wrist joint angles, wrist joint angle rates, and computed robot hand rotational rates as a function of time for resolved-rate control with rate limiting. At $t = 1.5$ sec, the robot wrist is positioned as shown in figure 9(b), which is the degenerate position. Note that if θ_5 were ever exactly 0, equation (15) would be indeterminant. The magnitudes of $\dot{\theta}_4$ and $\dot{\theta}_6$ are held at their maximum allowable value (which, in this paper, is $L = 3.5$ rad/sec) until θ_5 is such that computed speeds no longer have magnitudes which exceed this maximum speed value. While moving out of the degenerate region ($\theta_5 = 0°$), ω_X increases toward 1; but at the same time, ω_Y takes on values as large as 32 percent of the commanded value $(\omega_X)_c = 1$. This may result in undesirable motion of the robot hand, especially in the performance of a delicate task. After $\theta_5 \approx -0.23°$, the performance approaches the desired motion. Time is the reference parameter in table II but is probably not as important as accomplishing the desired movements of the robot hand. For example, it may not matter whether the robot hand reaches the position in figure 9(c) in 3 sec or 4 sec.

Using Generalized Matrix Inverse

In table III, the robot hand moves as commanded up to $t = 1.5$ sec. Thereafter, the robot hand rotates with $\omega_Z = -1$ deg/sec rather than with the commanded value $(\omega_X)_c = 1$ deg/sec.

Using Resolved-Rate Control With Coordinated Wrist

Figure 10 shows how the robot hand is maneuvered with the intermediate wrist movement, and table IV gives rotational information as a function of time. From $t = 0$ to $t = 1.5$ sec, the robot hand rotates to the vertical position ($\theta_5 = 0°$). From $t = 1.5$ sec to $t = 2.05$ sec, opposite rotations of θ_4 and θ_6 at maximum joint angle rates (or any assigned rates) locate the robot wrist for tilting to the desired position ($t = 3.55$ sec). There is a brief delay for the wrist to position itself (without moving the robot hand) in order to move the robot hand as commanded. For $\theta_5 = 0°$, there are no extraneous hand movements during the coordinated wrist movement, which, consequently, appears to enhance the dexterity of the robot arm and to provide a beneficial method of control.

CONCLUDING REMARKS

The robot wrist which orients the robot hand consists of three rotational joints. When two of these joints are colinear so that they produce identical rotations of the hand, a degree of freedom is lost, and a singular or degenerate condition exists. The usual approach of avoiding this degenerate wrist condition restricts the wrist workspace and requires additional task planning. This paper deals with methods to control the robot hand at or near this degenerate condition. The following comments apply in the degenerate region of the wrist joints:

1. Resolved-rate control results in excessive wrist joint angle rates. Setting the excessive rates equal to some maximum operational limit also leads to erratic behavior. For example, in some cases, the robot hand will not respond at all to commands. In other cases, a rate occurs about one hand axis when the command is for a rate about another axis.

2. The generalized matrix inverse can produce unwanted responses of the robot hand.

3. Resolved-rate control with a coordinated movement of the wrist near the wrist degenerate condition appears to enhance the dexterity of the robot arm. The method uses a coordinated movement of the first and third joints of the robot wrist to locate the second wrist joint axis for movement of the robot hand in the commanded direction. This method does not entail infinite joint angle rates or cause extraneous hand movements; however, there is a brief delay for the coordinated movement (which is accomplished in an optimal manner). In a delicate situation (for example, peg-in-the-hole task), this method of control may be beneficial. In addition to teleoperator systems, the coordinated movement can be implemented in higher level control systems. Also, with joint limits, there are orientations from which a

coordinated movement cannot be made. In these cases, the operator may choose to control the individual joint angles separately.

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APPENDIX A

HOMOGENEOUS TRANSFORMATION MATRICES

When the parameters in table I are introduced into the general transformation matrix in equation (1), the following six transformation matrices (which are the same as those used in ref. 4) result:

$$
A_0^1 = \begin{bmatrix} -\cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ -\sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & 1 & 0 & NO \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A1}
$$

$$
A_1^2 = \begin{bmatrix} -\sin\theta_2 & -\cos\theta_2 & 0 & -ES\sin\theta_2 \\ \cos\theta_2 & -\sin\theta_2 & 0 & ES\cos\theta_2 \\ 0 & 0 & 1 & SN \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A2}
$$

$$
A_2^3 = \begin{bmatrix} -\sin\theta_3 & 0 & \cos\theta_3 & 0 \\ \cos\theta_3 & 0 & \sin\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A3}
$$

$$
A_3^4 = \begin{bmatrix} -\cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ -\sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & 1 & 0 & WE \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A4}
$$

$$
A_4^5 = \begin{bmatrix} -\cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ -\sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A5}
$$

$$
A_5^6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & HW \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A6}
$$

APPENDIX B

ROTATIONAL VELOCITY OF ROBOT HAND IN ROBOT HAND AXIS SYSTEM

Additional symbols used in this appendix are defined as follows:

E_1, E_2, E_3 projection of rotational velocity attributed to joints 1, 2, and 3 along X_3, Y_3, and Z_3, respectively

Q_1, Q_2, Q_3, Q_4 expression for certain elements in R_6^3 defined by equations (B8), (B9), (B10), and (B11), respectively

The rotational velocity of the robot hand

$$\vec{\omega} = \vec{\omega}_{1,2,3} + \vec{\omega}_{4,5,6} \tag{B1}$$

is the summation of the individual contributions from the different joints in the robot arm. Expressing these contributions in the robot hand axis system is straightforward, but for convenience, some expressions for these contributions are contained in this appendix.

Rotational Velocity From Joints 1, 2, and 3

Rather than use equation (9) in the text, express the rotational velocity from joints 1, 2, and 3 in a more computationally efficient way as

$$\vec{\omega}_{1,2,3} = R_6^3 R_3^2 \left[R_2^1 \left(R_1^0 \left\{ \begin{matrix} 0 \\ 0 \\ \dot{\theta}_1 \end{matrix} \right\} + \left\{ \begin{matrix} 0 \\ 0 \\ \dot{\theta}_2 \end{matrix} \right\} \right) + \left\{ \begin{matrix} 0 \\ 0 \\ \dot{\theta}_3 \end{matrix} \right\} \right] \tag{B2}$$

Equation (B2) is partially expanded to

$$\vec{\omega}_{1,2,3} = R_6^3 \left\{ \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix} \right\} \tag{B3}$$

where

$$E_1 = -\dot{\theta}_1 \sin(\theta_2 + \theta_3) \tag{B4}$$

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$$E_2 = \dot{\theta}_2 + \dot{\theta}_3 \tag{B5}$$

$$E_3 = \dot{\theta}_1 \cos(\theta_2 + \theta_3) \tag{B6}$$

Then, with

$$R_6^3 = \begin{bmatrix} Q_1 & Q_2 & -\sin\theta_5 \cos\theta_6 \\ Q_3 & Q_4 & \sin\theta_5 \sin\theta_6 \\ \cos\theta_4 \sin\theta_5 & \sin\theta_4 \sin\theta_5 & \cos\theta_5 \end{bmatrix} \tag{B7}$$

where

$$Q_1 = \cos\theta_4 \cos\theta_5 \cos\theta_6 - \sin\theta_4 \sin\theta_6 \tag{B8}$$

$$Q_2 = \sin\theta_4 \cos\theta_5 \cos\theta_6 + \cos\theta_4 \sin\theta_6 \tag{B9}$$

$$Q_3 = -\cos\theta_4 \cos\theta_5 \sin\theta_6 - \sin\theta_4 \cos\theta_6 \tag{B10}$$

$$Q_4 = -\sin\theta_4 \cos\theta_5 \sin\theta_6 + \cos\theta_4 \cos\theta_6 \tag{B11}$$

equation (B3) becomes

$$\vec{\omega}_{1,2,3} = \left\{ \begin{matrix} Q_1 E_1 + Q_2 E_2 - \sin\theta_5 \cos\theta_6 \, E_3 \\ Q_3 E_1 + Q_4 E_2 + \sin\theta_5 \sin\theta_6 \, E_3 \\ \cos\theta_4 \sin\theta_5 \, E_1 + \sin\theta_4 \sin\theta_5 \, E_2 + \cos\theta_5 \, E_3 \end{matrix} \right\} \tag{B12}$$

Rotational Velocity From Joints 4, 5, and 6 (Wrist Joints)

Equation (10) in the text is better expressed as

$$\vec{\omega}_{4,5,6} = R_6^5 \left[R_5^4 \left(R_4^3 \left\{ \begin{matrix} 0 \\ 0 \\ \dot{\theta}_4 \end{matrix} \right\} + \left\{ \begin{matrix} 0 \\ 0 \\ \dot{\theta}_5 \end{matrix} \right\} \right) + \left\{ \begin{matrix} 0 \\ 0 \\ \dot{\theta}_6 \end{matrix} \right\} \right] \tag{B13}$$

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which expands to

$$
\vec{\omega}_{4,5,6} = \left\{ \begin{array}{c} -\dot{\theta}_4 \sin \theta_5 \cos \theta_6 + \dot{\theta}_5 \sin \theta_6 \\ \dot{\theta}_4 \sin \theta_5 \sin \theta_6 + \dot{\theta}_5 \cos \theta_6 \\ \dot{\theta}_4 \cos \theta_5 + \dot{\theta}_6 \end{array} \right\} \qquad (B14)
$$

Assumption

To simplify the analysis in this paper, assume that $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = 0$ so that $\vec{\omega}_{1,2,3} = 0$ in equation (B12). Hence, equation (B1) reduces to

$$
\vec{\omega} = \left\{ \begin{array}{c} \omega_X \\ \omega_Y \\ \omega_Z \end{array} \right\} = \left\{ \begin{array}{c} -\dot{\theta}_4 \sin \theta_5 \cos \theta_6 + \dot{\theta}_5 \sin \theta_6 \\ \dot{\theta}_4 \sin \theta_5 \sin \theta_6 + \dot{\theta}_5 \cos \theta_6 \\ \dot{\theta}_4 \cos \theta_5 + \dot{\theta}_6 \end{array} \right\} \qquad (B15)
$$

Equation (B15) is solved for the joint speeds as indicated by equations (14), (15), and (16) in the text.

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    Use With Kinematically Redundant Manipulators. IEEE Trans. Syst., Man, &
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TABLE I.- HOMOGENEOUS TRANSFORMATION MATRIX PARAMETERS

[From ref. 4]

Joint, i	α_i, deg	a_i	r_i	θ_i', deg	θ_i limits, deg
1	90	0	NO[*]	$\theta_1 + 180$	±160
2	0	ES[†]	SN[‡]	$\theta_2 + 90$	±165
3	90	0	0	$\theta_3 + 90$	±135
4	90	0	WE[§]	$\theta_4 + 180$	±135
5	90	0	0	$\theta_5 + 180$	±105
6	0	0	HW[‖]	θ_6	±270

[*]NO = Neck-to-base distance = 66.05 cm = 26 in.
[†]ES = Elbow-to-shoulder distance = 43.18 cm = 17 in.
[‡]SN = Shoulder-to-neck distance = 15.24 cm = 6 in.
[§]WE = Wrist-to-elbow distance = 43.15 cm = 17 in.
[‖]HW = Hand-to-wrist distance = 15.24 cm = 6 in. (In the kinematic equations, the distance from the hand axis system to the wrist is assumed to be 0.)

TABLE II.- RESOLVED-RATE CONTROL WITH RATE LIMITING

Time, t, sec	Wrist joint angle, deg			Wrist joint angle rate, deg/sec			Commanded hand rotational rate, deg/sec			Computed hand rotational rate, deg/sec		
	θ_4	θ_5	θ_6	$\dot\theta_4$	$\dot\theta_5$	$\dot\theta_6$	$(\omega_X)_c$	$(\omega_Y)_c$	$(\omega_Z)_c$	ω_X	ω_Y	ω_Z
0	0 →	1.5	0 →	0 →	-1 →	0 →	0 →	-1 →	0 →	0 →	-1 →	0 →
⋯		⋯										
1.5		≈0					→	→ 0		→	→	
1.6	18.05	-.02	-18.05	200.54 →	-.28	-200.54 →	1 →	0 →		.16	-.24	
1.7	38.10	-.07	-38.10		-.59					.55	-.32	
1.8	58.16	-.14	-58.16	86.36	-.83	-84.36				.97	-.02	
1.9	71.77	-.23	-71.77		-.95					1 →	.03	
⋯	⋯	⋯	⋯	⋯	⋯	⋯		→	→		⋯	→
3.0	86.97	-1.32	-86.97	2.32	-1	-2.32					0	
3.2	87.37	-1.52	-87.37	1.75	-1	-1.75				→	0	

TABLE III.— GENERALIZED MATRIX INVERSE CONTROL

Time, t, sec	Wrist joint angle, deg			Wrist joint angle rate, deg/sec			Commanded hand rotational rate, deg/sec			Computed hand rotational rate, deg/sec		
	θ_4	θ_5	θ_6	$\dot{\theta}_4$	$\dot{\theta}_5$	$\dot{\theta}_6$	$(\omega_X)_c$	$(\omega_Y)_c$	$(\omega_Z)_c$	ω_X	ω_Y	ω_Z
0	0	1.5	0	0	−1	0	0	−1	0	0	−1	0
···	→	···	→	→	→	→	→	→	→	→	→	→
1.5		0			0			0			0	
1.6	−.01	−.01		−1			1					−1
···	···	→		→	→		→	→			→	→
3.0	−1.49											

TABLE IV.- RESOLVED-RATE CONTROL WITH COORDINATED WRIST MOVEMENT

Time, t, sec	Wrist joint angle, deg			Wrist joint angle rate, deg/sec			Commanded hand rotational rate, deg/sec			Computed hand rotational rate, deg/sec		
	θ_4	θ_5	θ_6	$\dot{\theta}_4$	$\dot{\theta}_5$	$\dot{\theta}_6$	$(\omega_X)_c$	$(\omega_Y)_c$	$(\omega_Z)_c$	ω_X	ω_Y	ω_Z
0	0	1.5	0	0	-1	0	0	-1	0	0	-1	0
...	→	...	→	→	→	→	→	→	→	→	→	→
1.5		0						0			0	
1.5⁺				-200.54	0	200.54						
...		→		→	→	→	→	→	→	→	→	→
2.05	-90		90							1		
2.05⁺				0	1	0	1					
...	→	...	→	→	→	→	→	→	→	→	→	→
3.55		1.5										

Figure 1.- Joint axis systems defined by Denavit-Hartenberg parameters a_i, α_i, r_i, and θ_i'.
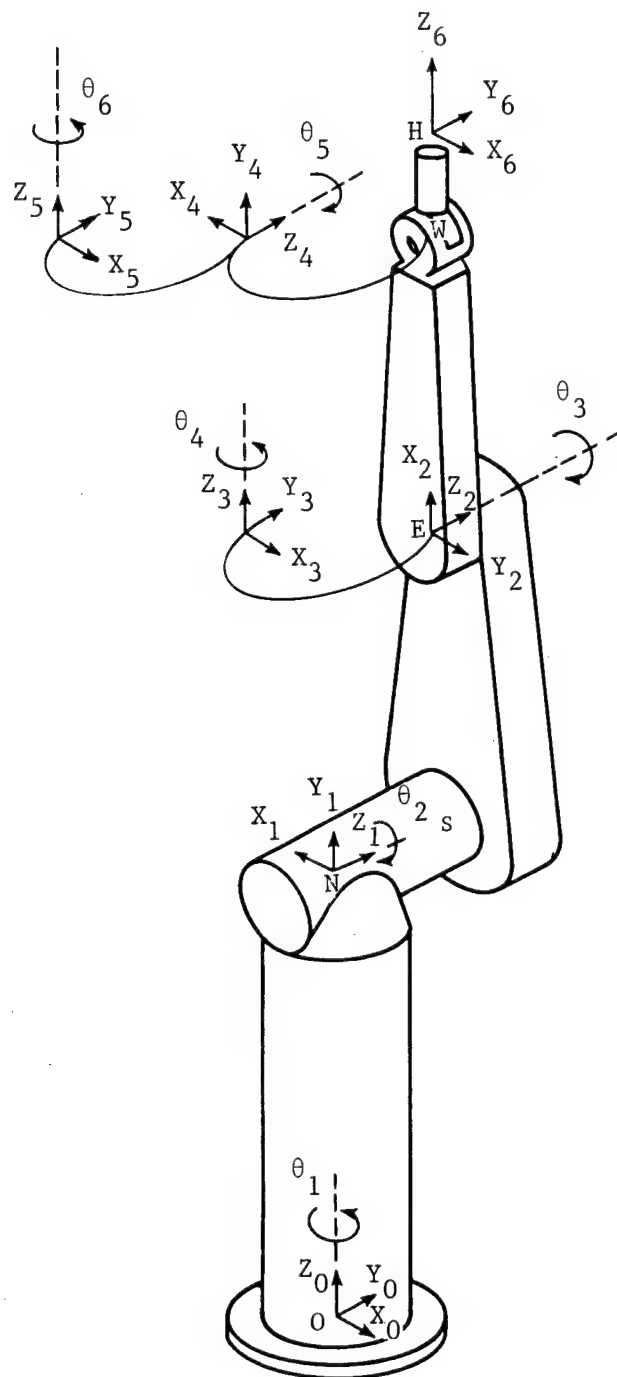
Figure 2.- Robot arm with axis systems.
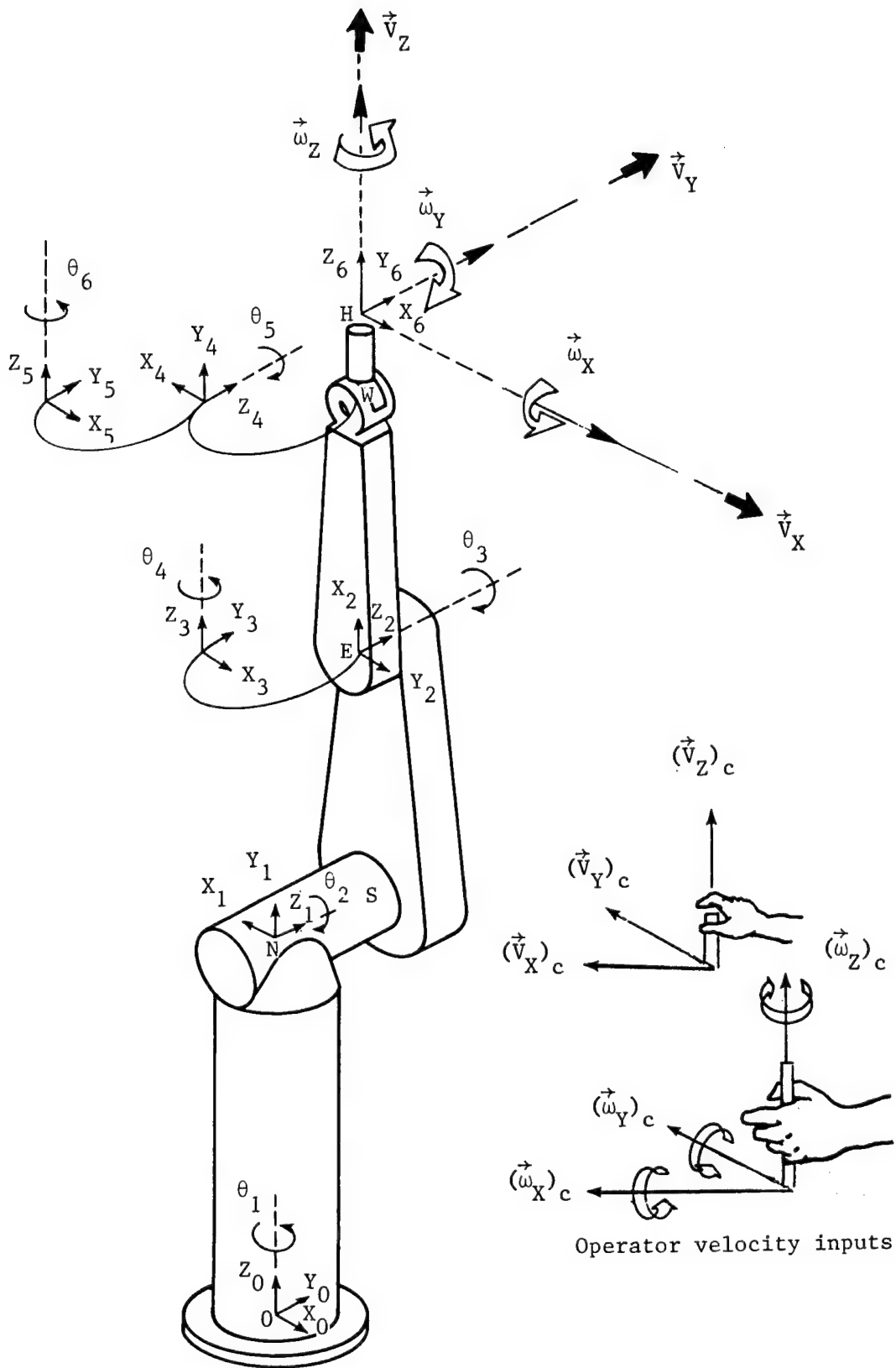
Figure 3.- Initial position of robot arm, joint axis systems, and
commanded robot hand velocities.
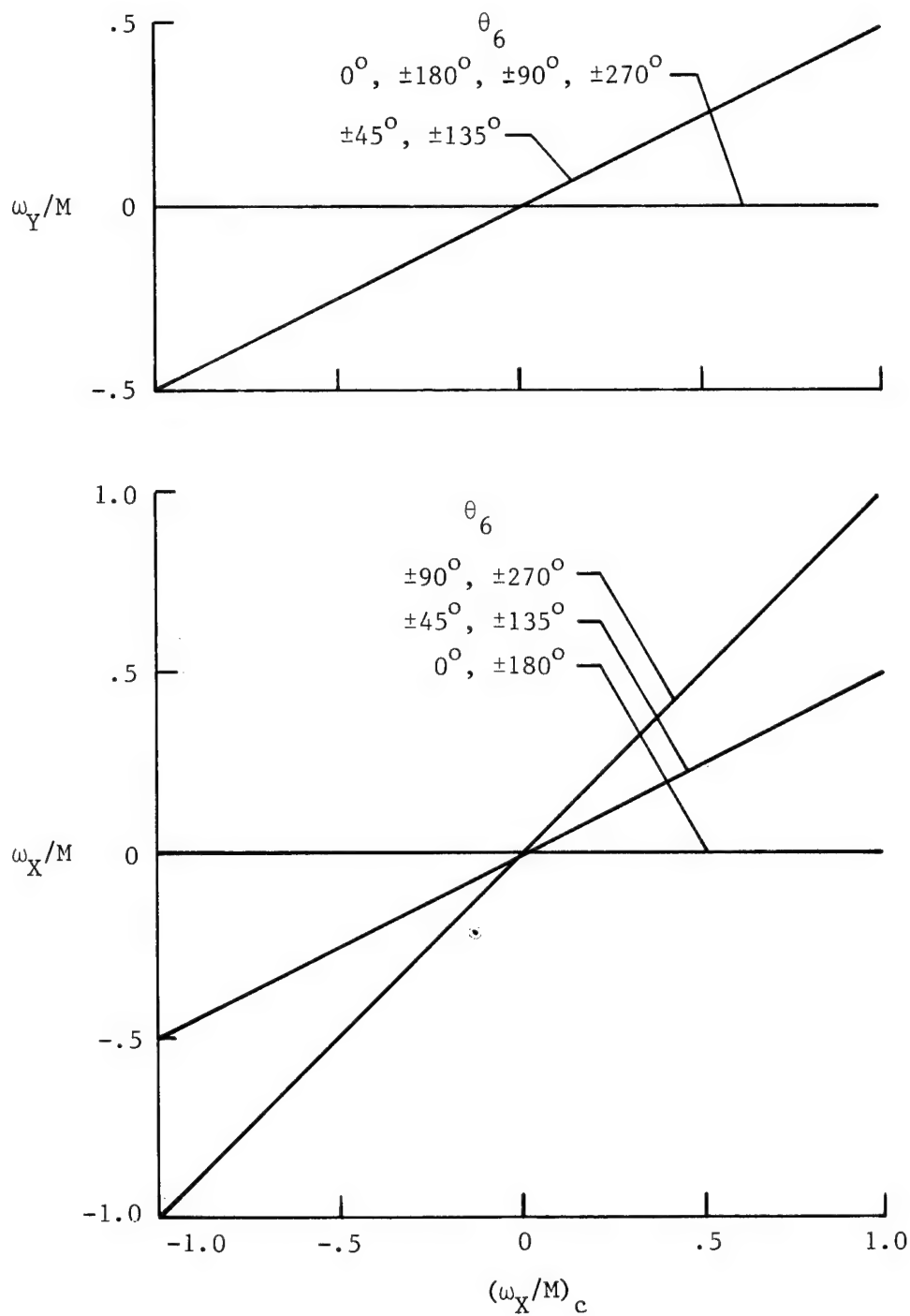
(a) $(\omega_Y/M)_C = 0.$

Figure 4.- Rotational velocity of robot hand versus commanded
velocities for resolved-rate control with rate limiting.
M is maximum angular wrist joint velocity; $\theta_5 = 0°.$
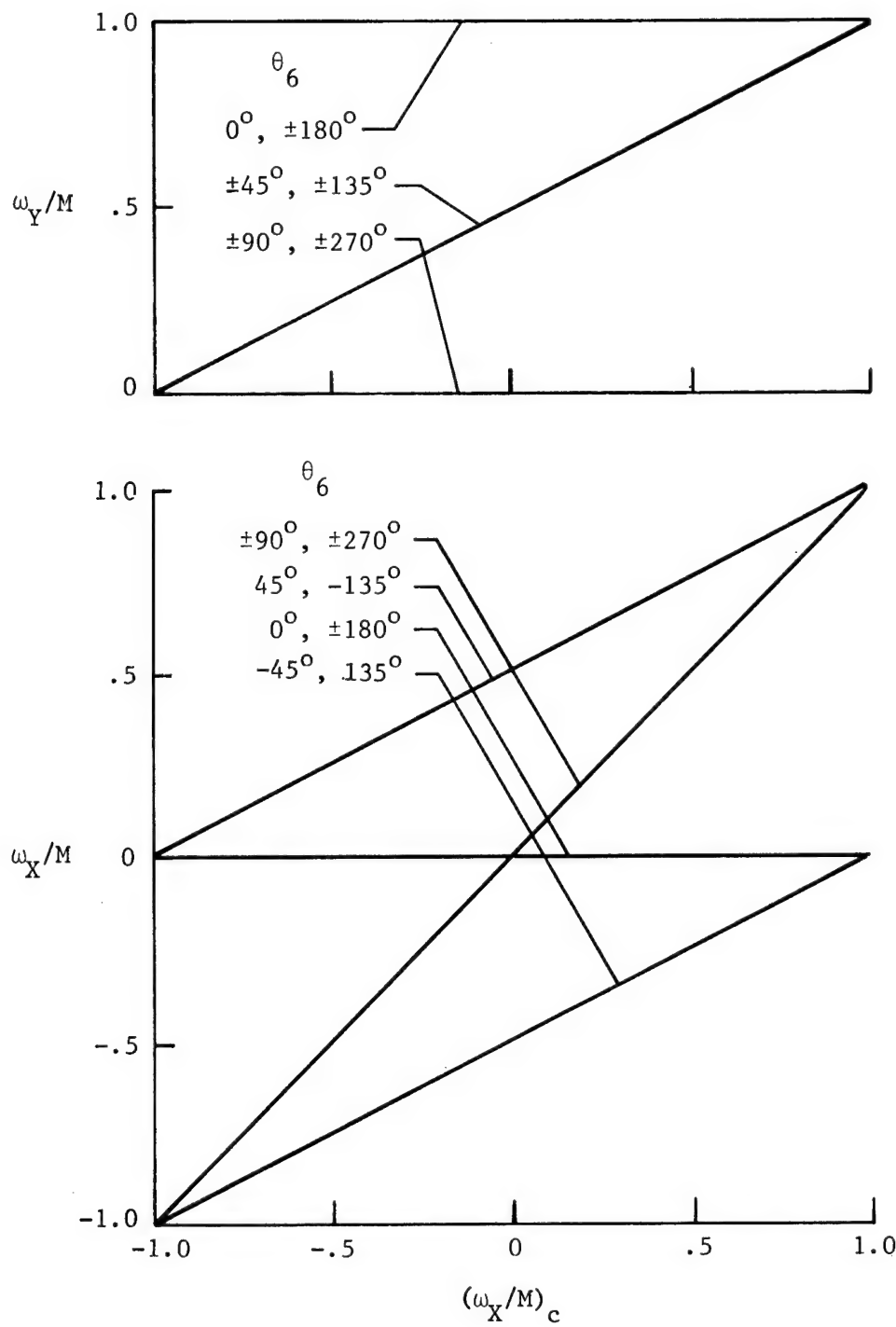
(b) $(\omega_Y/M)_c = 1.$

Figure 4.- Concluded.
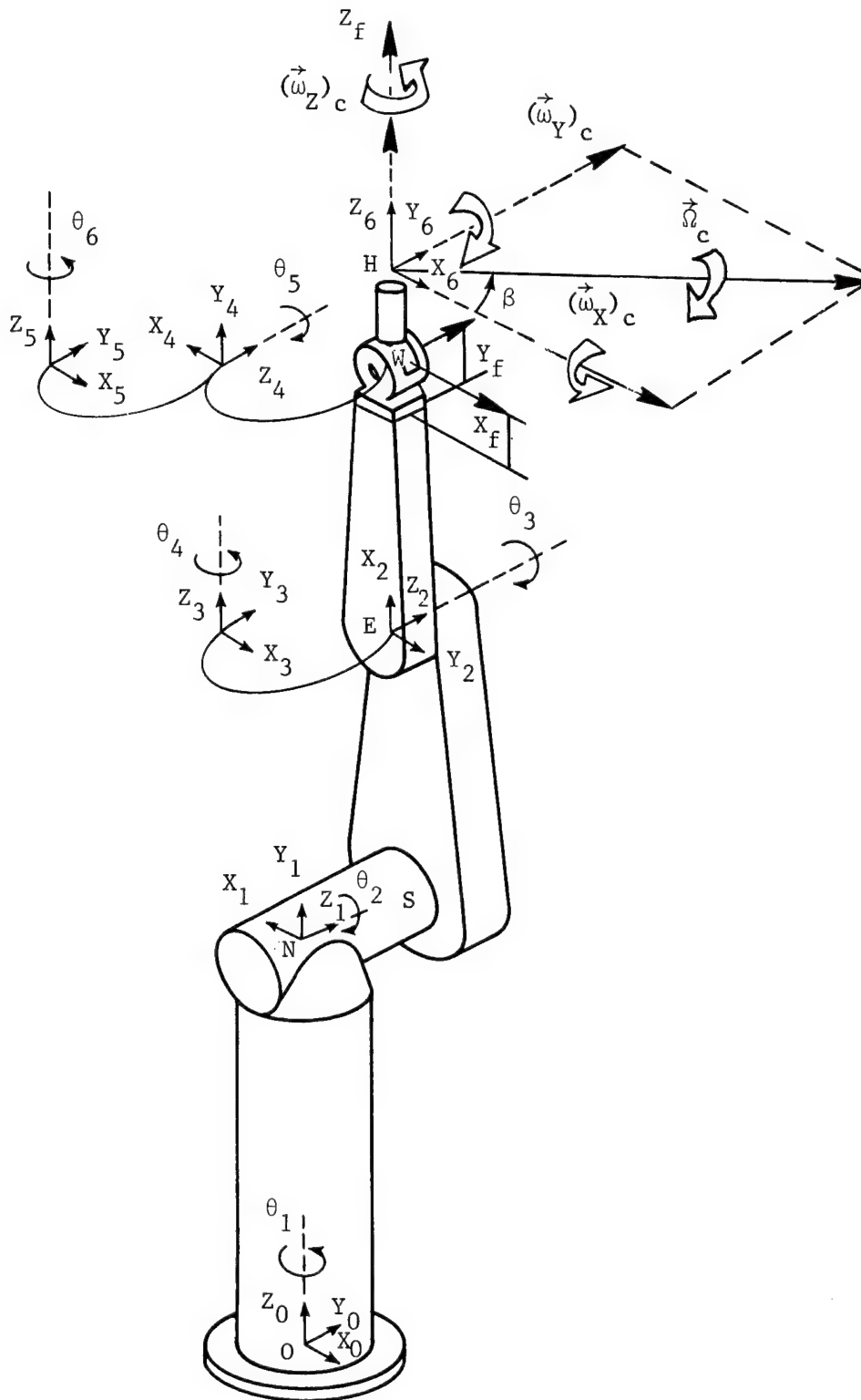
Figure 5.- Forearm axis system (X_f, Y_f, Z_f) and commanded rotational
velocity $\vec{\Omega}_c$ in plane of hand axes X_6 and Y_6.
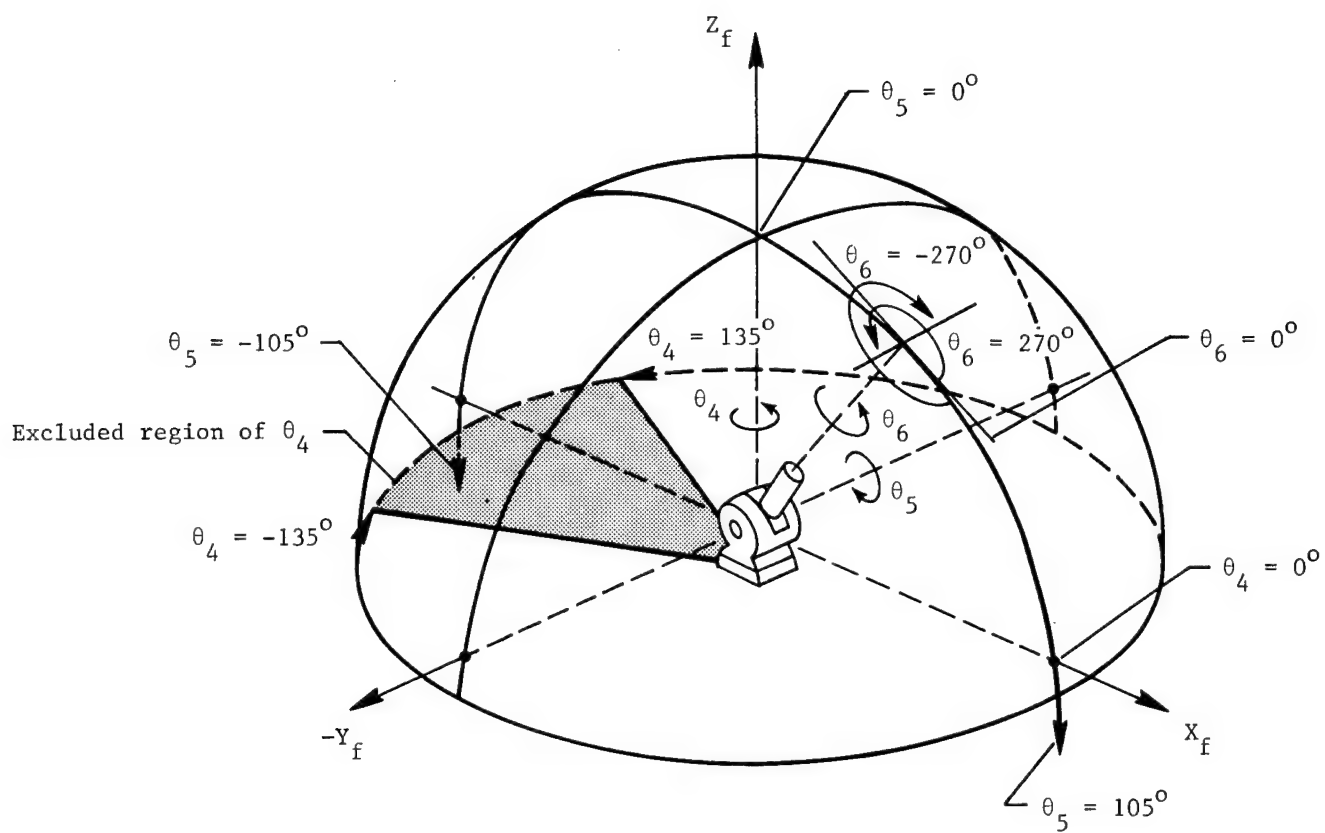
Figure 6.- Geometry showing wrist joint angle limits.
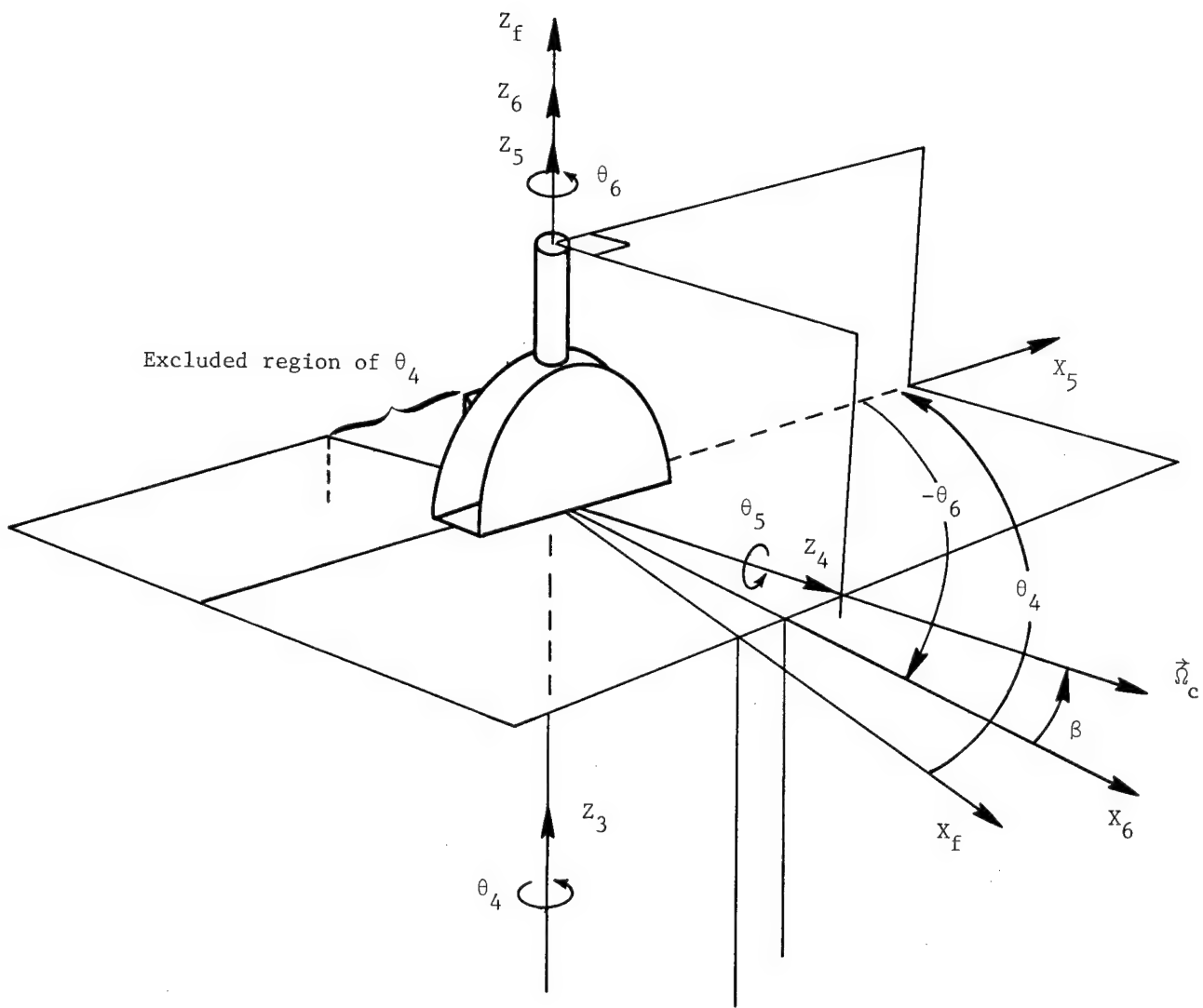
Figure 7.- Geometry involved in coordinated wrist movement.
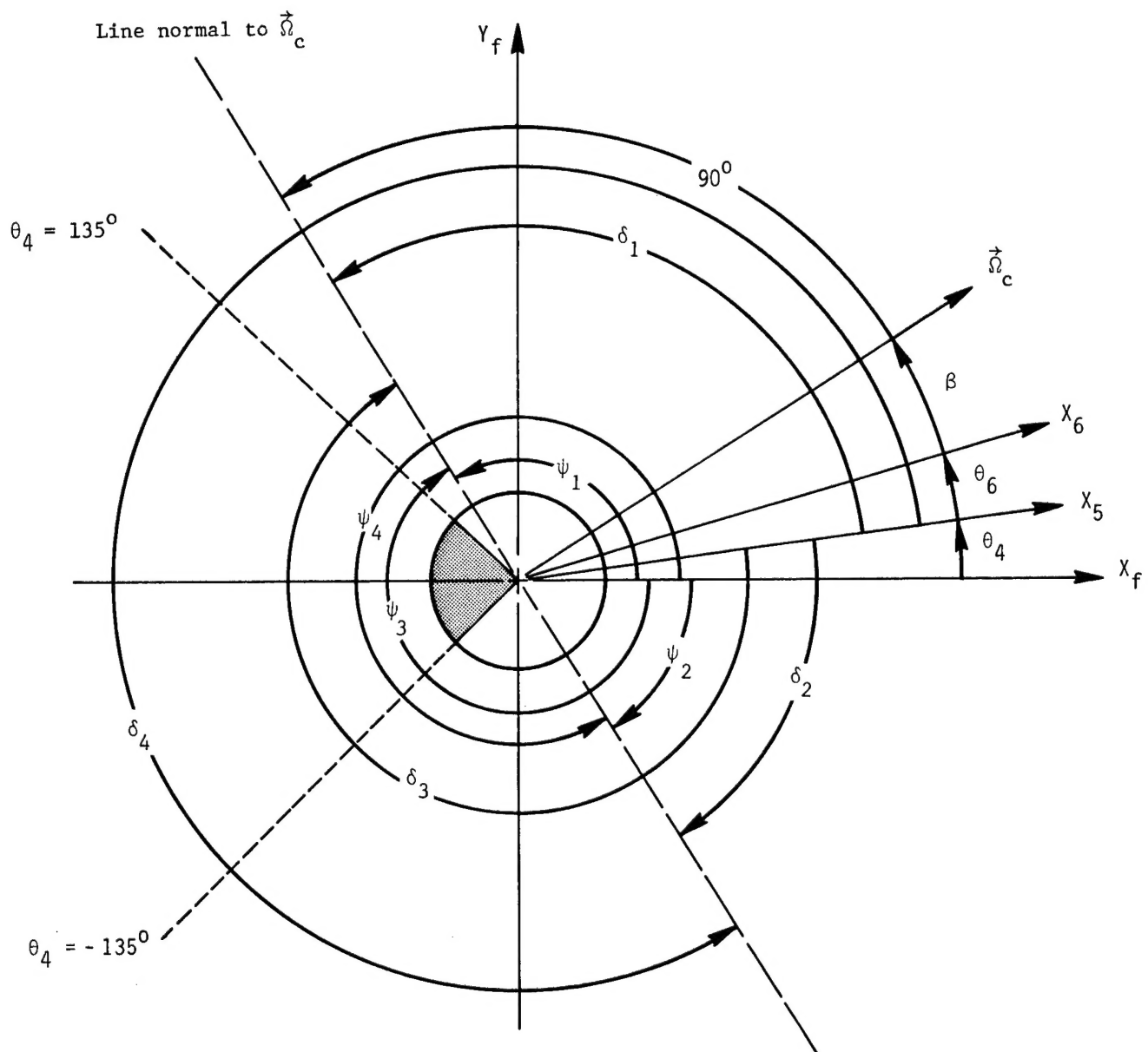
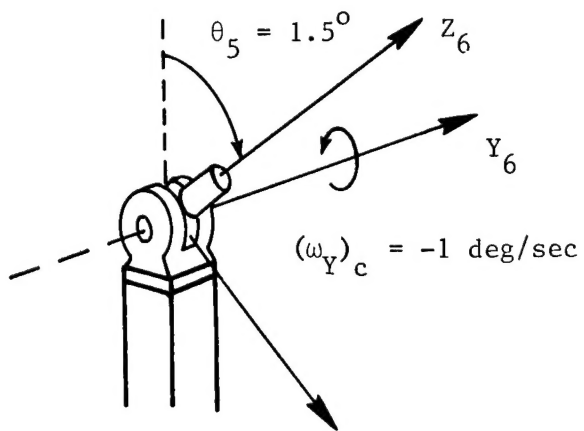Figure 8.- Angles used in choosing optimal coordinated movement.
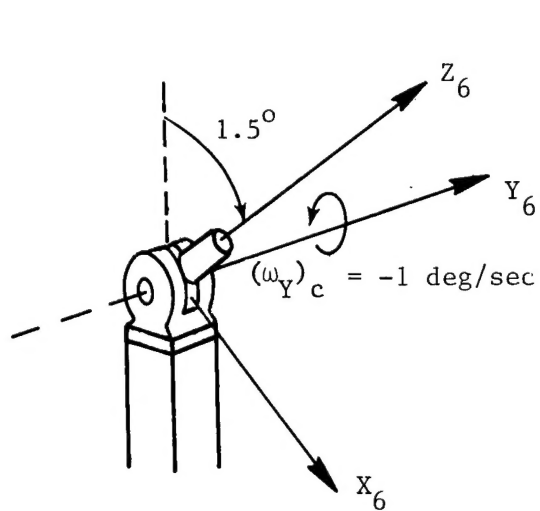
(a) Initial orientation.
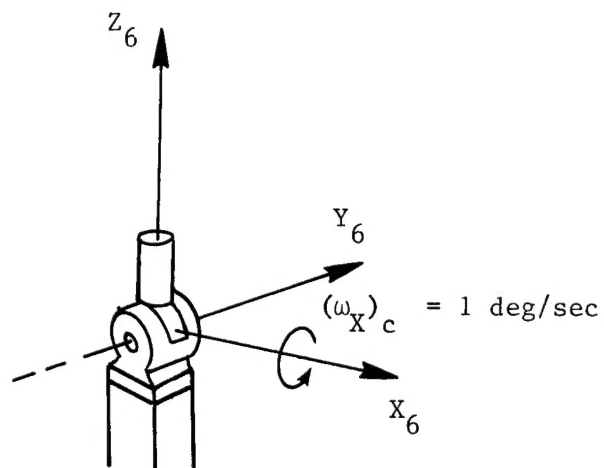
(b) Second orientation.
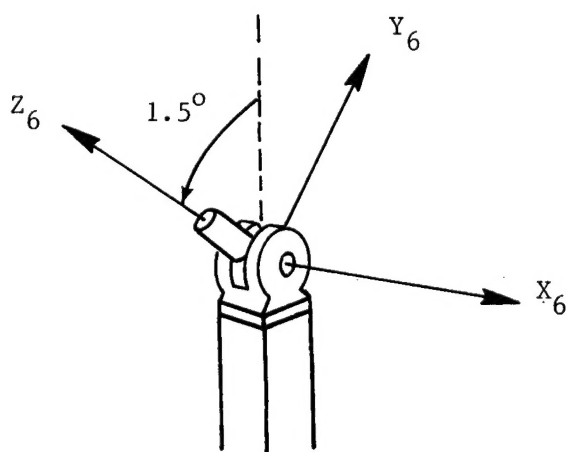
(c) Final orientation.

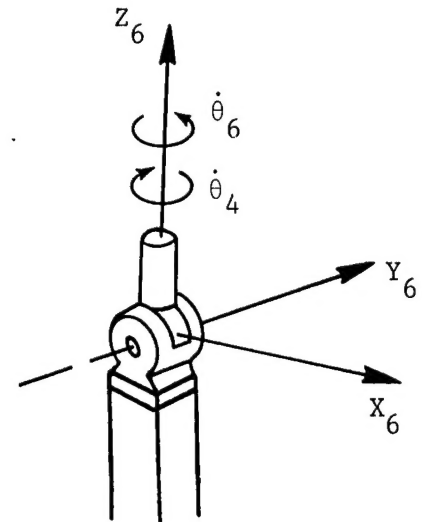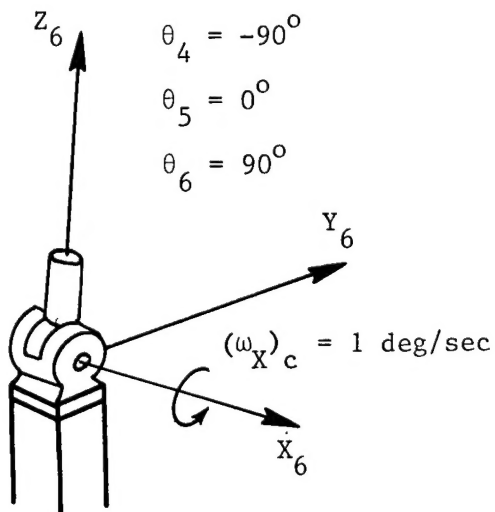Figure 9.– Three sequential wrist orientations and commanded rotational speeds in robot hand axis system.
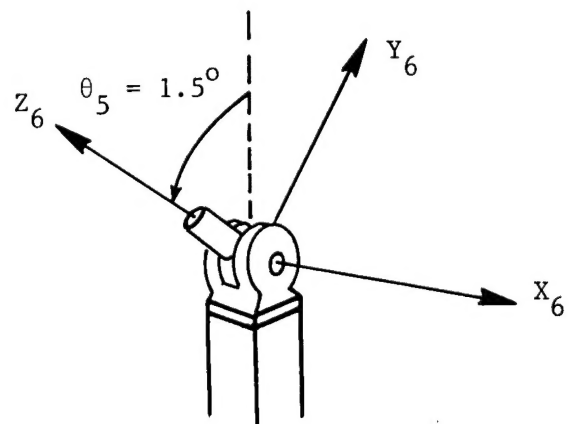
(a) Initial orientation.

(b) Second orientation and coordinated joint angle rates (equal magnitude but opposite directions).

(c) Intermediate orientation.

(d) Final orientation.

Figure 10.- Maneuvering robot hand axis system by using coordinated wrist rotation.

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16. Abstract

Kinematic resolved-rate equations allow an operator with visual feedback to dynamically control a robot hand. However, when the robot wrist is degenerate, the computed joint angle rates exceed operational limits, and unwanted hand movements can result. The generalized matrix inverse solution can also produce unwanted responses. In this paper, a new method is introduced to control the robot hand in the region of the degenerate robot wrist. The method uses a coordinated movement of the first and third joints of the robot wrist to locate the second wrist joint axis for movement of the robot hand in the commanded direction. There is a brief delay for the coordinated movement (which is accomplished in an optimal manner). The method does not entail infinite joint angle rates and appears to be a beneficial method of control.

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